Subject CM2

CMP Upgrade 2024/25

CMP Upgrade

This CMP Upgrade lists the changes to the Syllabus, Core Reading and the ActEd material since last year that might realistically affect your chance of success in the exam. It is produced so that you can manually amend your 2024 CMP to make it suitable for study for the 2025 exams. It includes replacement pages and additional pages where appropriate.

Alternatively, you can buy a full set of up-to-date Course Notes / CMP at a significantly reduced price if you have previously bought the full-price Course Notes / CMP in this subject. Please see our 2025 *Student Brochure* for more details.

We only accept the current version of assignments for marking, *ie* those published for the sessions leading to the 2025 exams. If you wish to submit your scripts for marking but only have an old version, then you can order the current assignments free of charge if you have purchased the same assignments in the same subject in a previous year, and have purchased marking for the 2025 session.

This CMP Upgrade contains:

- all significant changes to the Syllabus and Core Reading
- additional changes to the ActEd Course Notes and Assignments that will make them suitable for study for the 2025 exams.

1 Changes to the Syllabus

This section contains all the ***non-trivial*** changes to the syllabus objectives.

The following objectives have been removed:

- 3.6.5 Generalisation of the two-state model to the JLT.
- 5.2.6 Use the martingale approach to pricing and hedging using the binomial model.

2 Changes to the Core Reading and ActEd material

This section contains all the ***non-trivial*** changes to the Core Reading and ActEd text.

Chapter 9

Section 3

New material has been written to cover the lognormal model in discrete time. The additional pages can be found at the end of this document.

Chapter 14

Sections 2.2 & 3

Material covering the 5-step method proof in discrete time been removed.

Chapter 17

Sections 6 & 7

Material covering the JLT model has been removed.

3 Changes to the X Assignments

Overall

There have been minor changes throughout the assignments.

More significant changes are listed below.

Assignment X4

Question X4.2 has been replaced by the following:

A bank is using a two-state discrete-time Markov chain model to value its bond portfolio.

On 1 January each year the bank assigns each of its client companies to one of the following categories:

- State F: The bank receives any payments due that year in full.
- State H: The bank receives only 50% of any payments due that year.

The diagram shows the risk-neutral probabilities that each company will move from its current rating level to the other level at the time of each review. These probabilities are independent of the company's previous ratings and the behaviour of other companies.

Let $p_{ij}(s,t)$ denote the probability that a company in State i at time s will be in State j at time t.

(i) Calculate $p_{FF}(0,t)$ and $p_{FH}(0,t)$ for $t=1,2,3$. [3]

The bank is considering purchasing at par a 3-year bond issued by a company currently rated as F. Under the terms of the bond, interest of 10% of the face value of the bond will be paid at the end of each year, and the bond will be redeemed at par at the end of the 3 years.

The annual effective yields on 1-year, 2-year and 3-year government bonds are all 5%.

(ii) (a) Calculate the risk-neutral expected present value of the payments from the bond per £100 face value.

(b) Comment on your answer in (ii)(a). [4]

After negotiations, the bank agrees to purchase the bonds at a price of £95.20.

(iii) Calculate the credit spread for this bond. [3]

[Total 10]

4 Changes to the Y Assignments

Overall

There have been minor changes throughout the assignments.

5 Changes to the Mock Exam

Overall

There have been minor changes throughout the Mock Exam.

More significant changes are listed below.

Question 9

The question wording has been altered for clarity in parts (v) and (vi):

An alternative approach is for the insurer to transfer a quarter of its existing policies to a reinsurance company for a single fee of £90,000 (*ie* the insurer pays the reinsurer a fee to take the policies).

(v) Calculate the probability of ruin over the next year if the insurer makes this transfer and comment on the result. [4]

(vi) Determine the maximum fee the insurer would be willing to pay the reinsurer to reduce the probability of ruin. [4]

6 Other tuition services

In addition to the CMP, you might find the following services helpful with your study.

6.1 Study material

For further details on ActEd's study materials, please refer to the *Products* pages on the ActEd website at **ActEd.co.uk**.

6.2 Tutorials

We offer the following (face-to-face and/or online) tutorials in Subject CM2:

- a set of Regular Tutorials (lasting a total of five days)
- a Block (or Split Block) Tutorial (lasting five full days)
- an Online Classroom.

For further details on ActEd's tutorials, please refer to our latest *Tuition Bulletin*, which is available from the ActEd website at **ActEd.co.uk**.

6.3 Marking

You can have your attempts at any of our assignments or mock exams marked by ActEd. When marking your scripts, we aim to provide specific advice to improve your chances of success in the exam and to return your scripts as quickly as possible.

For further details on ActEd's marking services, please refer to the 2025 *Student Brochure*, which is available from the ActEd website at **ActEd.co.uk**.

6.4 Feedback on the study material

ActEd is always pleased to receive feedback from students about any aspect of our study programmes. Please let us know if you have any specific comments (*eg* about certain sections of the notes or particular questions) or general suggestions about how we can improve the study material. We will incorporate as many of your suggestions as we can when we update the course material each year.

If you have any comments on this course, please send them by email to **CM2@bpp.com**.

3 The discrete-time lognormal model

Suppose that an investor wishes to invest a lump sum into a fund with compound investment rate growth for *n* years. This investment return is not known *now*, but will be determined immediately after the investment has been made. The accumulated value of the sum will, of course, be dependent on the investment rate.

Measuring time in years, consider the time interval $[0,n]$ subdivided into successive periods $[0,1], [1,2], \ldots, [n-1,n]$. For $t=1,2,\ldots,n$ let i_t be the investment return obtainable over the *t*th year, *ie* the period $[t-1,t]$. A single investment of 1 at time 0 will accumulate at time *n* to:

$$S_n = (1+i_1)(1+i_2)\ldots(1+i_n)$$

In general a theoretical analysis of the distribution function for S_n is somewhat difficult, even in the relatively simple situation when the yields each year are independent and identically distributed. There is, however, one special case for which an exact analysis of the distribution function for S_n is particularly simple.

Due to the compounding effect of investment returns, the accumulated value of an investment bond grows multiplicatively. This makes the lognormal distribution a natural choice for modelling the annual growth factors $1+i$, since a lognormal random variable can take any positive value and has the following multiplicative property:

If $X_1 \sim \log N(\mu_1, \sigma_1^2)$ and $X_2 \sim \log N(\mu_2, \sigma_2^2)$ are independent random variables, then:

$$X_1 X_2 \sim \log N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$$

Suppose that the random variable $\log(1+i_t)$ is normally distributed with mean μ and variance σ^2. In this case, the variable $(1+i_t)$ is said to have a lognormal distribution with parameters μ and σ^2.

So, if $\log(1+i_t) \sim N(\mu, \sigma^2)$, then $1+i_t \sim \log N(\mu, \sigma^2)$.

The above equation for S_n is therefore equivalent to:

$$\log S_n = \sum_{t=1}^{n} \log(1+i_t)$$

The sum of a set of independent normal random variables is itself a normal random variable. Hence, when the random variables $(1+i_t)$ $(t \geq 1)$ are independent and each has a lognormal distribution with parameters and μ and σ^2, the random variable S_n has a lognormal distribution with parameters $n\mu$ and $n\sigma^2$.

Since the distribution function of a lognormal variable is readily written down in terms of its two parameters, in the particular case when the distribution function for the yield each year is lognormal we have a simple expression for the distribution function of S_n.

So $S_n \sim \log N(n\mu, n\sigma^2)$ or $\log S_n \sim N(n\mu, n\sigma^2)$, and the distribution function of S_n can therefore be written:

$$P(S_n \le s) = \Phi\left(\frac{\log s - n\mu}{\sigma\sqrt{n}}\right)$$

Question

An individual now aged exactly 50 has built up a savings fund of £400,000. In order to retire at age 60, they will require a fund of at least £600,000 at that time. The annual returns on the fund, i, are independent and identically distributed, with $1+i \sim \log N(0.075, 0.1^2)$.

Calculate the probability that, if no further contributions are made to the fund, they will be able to retire at age 60.

Solution

If no further contributions are made, then the accumulated fund at age 60 will be $400{,}000 S_{10}$.

So, the probability that the fund will be sufficient to retire is:

$$\begin{aligned} P(400{,}000 S_{10} \ge 600{,}000) &= 1 - P\left(S_{10} \le \frac{600{,}000}{400{,}000}\right) \\ &= 1 - \Phi\left(\frac{\log 1.5 - 10\mu}{\sigma\sqrt{10}}\right) \\ &= 1 - \Phi(-1.0895) = \Phi(1.0895) = 0.862 \end{aligned}$$

Similarly for the present value of a sum of 1 due at the end of n years:

$$V_n = (1+i_1)^{-1} \ldots (1+i_n)^{-1}$$

$$\Rightarrow \qquad \log V_n = -\log(1+i_1) - \ldots - \log(1+i_n)$$

Since, for each value of t, $\log(1+i_t)$ is normally distributed with mean μ and variance σ^2, each term on the right-hand side of the above equation is normally distributed with mean $-\mu$ and variance σ^2. Also the terms are independently distributed. So, $\log V_n$ is normally distributed with mean $-n\mu$ and variance $n\sigma^2$. That is, V_n has lognormal distribution with parameters $-n\mu$ and $n\sigma^2$.

So $V_n \sim \log N(-n\mu, n\sigma^2)$ or $\log V_n \sim N(-n\mu, n\sigma^2)$, and the distribution function of V_n can therefore be written:

$$P(V_n \le s) = \Phi\left(\frac{\log s - (-n\mu)}{\sigma\sqrt{n}}\right) = \Phi\left(\frac{\log s + n\mu}{\sigma\sqrt{n}}\right)$$

By statistically modelling S_n and V_n, it is possible to answer questions such as:

- to a given point in time, for a specified confidence interval, what is the range of values for an accumulated investment?
- what is the maximum loss which will be incurred with a given level of probability?

It can also be noted that these techniques may be extended to calculate the risk metrics such as VaR, as introduced in a previous chapter, of a series of investments.

Question

The annual returns on a fund, i , are independent and identically distributed. Each year, the distribution of $1+i$ is lognormal with parameters $\mu = 0.075$ and $\sigma^2 = 0.025^2$.

Calculate the upper and lower quartiles for the accumulated value at the end of 5 years of an initial investment of £1,000.

Solution

By definition, the accumulated amount $1{,}000S_5$ will exceed the upper quartile u with probability 25%, *ie*:

$$0.75 = P(1{,}000S_5 \le u) = P(S_5 \le u/1{,}000)$$

So, using the formula for the distribution function of S_5:

$$0.75 = P(S_5 \le u/1{,}000) = \Phi\left(\frac{\log(u/1{,}000) - 5\mu}{\sigma\sqrt{5}}\right)$$

From page 162 of the *Tables*, we find that $\Phi(0.6745) = 0.75$. So, we must have:

$$\frac{\log(u/1{,}000) - 5\mu}{\sigma\sqrt{5}} = 0.6745 \quad ie \quad u = 1{,}000e^{5\mu + 0.6745\sigma\sqrt{5}} = £1{,}511$$

Similarly, the lower quartile is:

$$l = 1{,}000e^{5\mu - 0.6745\sigma\sqrt{5}} = £1{,}401$$